SPECIAL FUNCTIONS AND COMPLEX VARIABLES
(Common to ECE & EEE)

Course Code: 13BM1104

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Pre requisites

- Basic Knowledge in evaluation of definite integrals.
- Calculus of functions of real variables.

Course Educational Objectives:

- The aim of this course is to introduce the special functions, their generating functions and the algebra, geometry and calculus of functions of a complex variable.
- The emphasis will be on gaining a geometric understanding of complex analytic functions as well as developing computational skills in employing the powerful tools of complex analysis for solving theoretical and applied problems.

Course Outcomes:

Upon successful completion of the course, the students should be able to

- Evaluate improper integrals using beta and gamma functions.
- Use elementary analytic functions like the exponential and logarithmic functions, trigonometric functions.
- Use residue calculations as integration method and find the Taylor or Laurent series of a given function.
- Determine basic mapping properties of elementary functions, including how functions transform simple shapes in the plane such as lines and circles.
- Apply mathematical reasoning and the theory of complex variables to solve theoretical and applied problems.
UNIT-I (12 Lectures)

SPECIAL FUNCTIONS-1: (Beta, Gamma and Legendre functions)
Beta-function, Gamma function, Relation between Beta and Gamma functions, Series solution of Legendre’s equation, Legendre’s function, Rodrigue’s formula, Legendre polynomials, Generating function, Recurrence formulae, Orthogonality of Legendre Polynomials, Fourier-Legendre expansion of $f(x)$. (7.14 - 7.16, 16.13 - 16.17)

UNIT-II (12 Lectures)
Special functions-2 (Bessel function)
Bessel’s equation, Bessel’s function, Recurrence formulae for Bessel function $J_n(x)$, Expansions for $J_0$ and $J_1$, value of $J_\frac{1}{2}(x)$, Generating function for $J_n(x)$, Orthogonality of Bessel’s function, The Strum-Liouville problem: Eigen Values, Eigen functions and Orthogonality of eigen functions. (16.5-16.9, 16.1, 16.19)

UNIT-III (12 Lectures)
FUNCTIONS OF A COMPLEX VARIABLE:
Complex function, Real and Imaginary parts of Complex function, Limit, Continuity and Derivative of complex function, Cauchy-Riemann equations, Analytic function, entire function, singular point, conjugate function, equations in polar form, Harmonic functions, Milne-Thomson method, Simple applications to flow problems, Line integral of a complex function, Cauchy’s theorem (only statement), Cauchy’s Integral Formula. (19.7, 19.12, 20.2-20.6, 20.12-20.14)

UNIT-IV (12 Lectures)
SERIES OF COMPLEX TERMS AND RESIDUES:
Absolutely convergent and uniformly convergent of series of complex terms, Radius of convergence, Taylor’s series, Maclaurin’s series expansion, Laurent’s series. Zeros of an analytic function, Singularity, Isolated singularity, Removable singularity, Essential singularity, pole of order m,
simple pole, Residues, Residue theorem, Calculation of residues, Residue at a pole of order m, Evaluation of real definite integrals: Integration around the unit circle, Integration around semi circle, Indenting the contours having poles on the real axis.
(20.16-20.19, 20.20(a),(b),(d))

UNIT-V  (12 Lectures)

CONFORMAL TRANSFORMATION:
Standard transformations: Translation, Magnification and rotation, Inversion and reflection, Bilinear transformation, Properties, Conformal transformation, critical point, fixed points of a transformation, Special Conformal transformations: (20.8-20.10)

TEXT BOOK:

REFERENCES: